XXX. Appendix to the Account of the Earthquake-Wave Experiments made at Holyhead.

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I am now enabled to fulfill the intention expressed (vol. cli. p. 678) in concluding the above 'Account,' as read to the Royal Society, having since then completed a series of experiments upon the compression of specimens of the Holyhead Rocks, and determined their moduli of elasticity. These experiments were made upon cubes cut from solid and perfect pieces of the rocks by the lapidary's wheel, each 0.707 inch upon the edge—each side, therefore, presenting a surface of 0.5 square inch,—and the utmost care being taken to preserve perfect parallelism between the opposite boundary planes, so that, when compressed between hardened steel surfaces, fracture should not result by inequality of pressure.

The compressions were made at the Royal Arsenal, Woolwich, with the very accurate and excellent machine used for testing compression and extension of metals in the gun-factory; and I have to express my thanks to Lieut.-Col. Anderson, C.E., the Superintendent of that department, for the valuable assistance afforded me through his attention.

The specimens operated on consisted of two each from the following four classes, namely, the hardest and the softest slate-rock, the hardest and the softest quartz-rock, which occur within the range or neighbourhood of my experimental explosions at Holyhead; and from each of these classes or varieties of the two rocks, cubic specimens were compressed, 1st, in a direction transverse to the plane of lamination, 2nd, parallel to the same, all the cubes being so cut out of the rock that two sides were, quam prox., parallel to the plane of natural lamination or jointing. The load (50 lbs.) first applied was considered zero, being only sufficient to ensure a complete bearing in all parts of the instrument. The subsequent loads advanced by 1000 lbs. per square inch of surface at a time, up to the crushing of the specimen; and at each fresh load the amount of compression was measured by beam-callipers, with instrumental arrangements that admitted of reading space to .0005 of an inch.

The experimental results, as obtained, are recorded in the following Tables, from No. I. to No. VIII. inclusive; and in the succeeding Tables IX. and X., the results of the former are compared, and the mean compression deduced for each 1000 lbs. of pressure applied upon a prism of each of the four classes of rock (two of slate and two of quartz), of one inch square surface, and one inch in height, and under both conditions as to the relative direction of pressure and of lamination.

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Table I.—Holyhead-Rock Compression. Experiments A, on Hard Slate; pressure transverse to lamination.



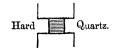
No. of experiment.	Pressure due to the unit of surface =1 square inch.	the unit of readings of the surface column of		Total compressions produced by the load on column of 0.707 inch.	Total compressions reduced to a column of unit height = 1 inch.		
	lbs.	in.	in.	in.	in.		
1.	50	.085	.000	.000	.0000		
2.	1,000	.081	.004	.004	.0052		
3.	2,000	·078+	·003 —	.007	.0091		
4.	3,000	·078+	.000	.007	.0091		
5.	4,000	·078	·000+	.007	·0091		
6.	5,000	·078	.000	•007	•0091		
7.	6,000	·077+	·001—	.007	•0091		
8.	7,000	.077	+000	•008	.0104		
9.	8,000	·076+	.001 —	•008	.0104		
10.	9,000	·076+	•000	•008	.0104		
11.	10,000	·076+	•000	•008	.0104		
12.	11,000	076	·000+	•008	·0104		
13.	12,000	•076	.000	•009	•0117		
14.	13,000	·075+	·001 —	•009	•0117		
15.	14,000	$\cdot 075 +$.000	•009	•0117		
16.	15,000	·075+	.000	•009	•0117		
17.	16,000	.075	+000	•009	.0117		
18.	17,000	.075	•000	.009	·0117		
19.	18,000	.075	.000	•009	.0117		
20.	19,000	·075	.000	.010	.0130		
21.	20,000	$\cdot 074 +$	·001—	.010	.0130		
22.	21,000	$\cdot 074 +$.000	.010	.0130		
23.	22,000	.074	•000 +	•010	.0130		
24.	23,000	.074	•000	•011	.0143		
25.	24,000	Crushed.		•011	.0143		

Table II.—Holyhead-Rock Compression. Experiments B, on Hard Slate; pressure parallel to lamination.

Hard		Slate.
	1 1	

1.	50	·130	.000	.000	.0000
2.	1,000	.120	.010	.010	.0130
3.	2,000	.100	.020	.030	.0390
4.	3,000	·099+	·001 —	·031+	·0403+
5.	4,000	•098	·001+	.032	.0416
6.	5,000	.097	.001	.032	.0416
7.	6,000	•096	•001	.032	.0416
8.	7,000	•094	.002	•036	.0468
9.	8,000	·092+	·002—	·038+	•0494
10.	9,000	·092+	.000	·038+	.0494
11.	10,000	·092+	.000	·038+	.0494
12.	11,000	.092	·000+	·038+	.0494
13.	12,000	•092	.000	·038+	•0494
14.	13,000	•092	.000	•038+	.0494
15.	14,000	.092	.000	·038+	.0494
16.	15,000	•090	.002	.040	.0520
17.	16,000	.089	•001	•041	.0533
18.	17,000	•086	.003	.044	.0572
19.	18,000	·085+	·001 —	·045+	·0585+
20.	19,000	·085+	.000	·045+	·0585+
21.	20,000	·085+	.000	·045+	·0585+
22.	21,000	.085	·000+	·045+	·0585+
23.	22,000	·085	.000	·045+	·0585+
24.	23,000	.085	.000	·045+	·0585+-
25.	24,000	•082	.003	.048	.0624
26.	25,000	.082	.000	.048	•0624
27.	26,000	•080	.002	.050	•0650
28.	27,000	.077	.003	.053	•0689
29.	27,000+	Crushed.		.053	•0689

Table III.—Holyhead-Rock Compression. Experiments C, on Hard Quartz; pressure transverse to lamination.



No. of experiment.	Pressure due to the unit of surface = 1 square inch.	Compression readings of the column of 0.707 inch.	Compression readings due to the successive loads.	Total compressions produced by the load on column of 0.707 inch.	Total compressions reduced to a column of unit height = 1 inch.
	lbs.	in.	in.	in.	in.
1.	50	•100	•000	•000	•0000
2.	1,000	· 0 97	.003	•003	•0039
3.	2,000	·095+	·002—	.003	.0039
4.	3,000	·095+	•000	.003	•0039
5.	4,000	·095+	•000	.003	.0039
6.	5,000	·095+	•000	.003	•0039
7.	6,000	•095	·000 +	•003	.0039
8.	7,000	•095	•000	.003	•0039
9.	8,000	•095	•000	•005	.0065
10.	9,000	•094	•001	•006	.0078
11.	10,000	·093+	·001 —	•006	.0078
12.	11,000	·093+	. •000	.006	.0078
13.	12,000	·093+	•000	•006	.0078
14.	13,000	•093	·000+	•006	•0078
15.	14,000	•093	•000	•006	.0078
16.	15,000	•093	•000	•006	.0078
17.	16,000	•093	•000	.007	•0091
18.	17,000	·092+	·001—	•007	•0091
19.	18,000	.092	·000+	.007	•0091
20.	19,000	•092	•000	•008	.0104
21.	20,000	·091+	·001	·009+	·0117+
22.	21,000	•088	·003+	.012	•0156
23.	22,000	·083+	·005—	•012	•0156
24.	23,000	·083+	•000	.012	•0156
25.	24,000	·083+	•000	.012	·0156
26.	25,000	•083	·000+	.012	•0156
27.	26,000	•083	.000	•017	•0221
28.	27,000	·082+	·001 —	·017	.0221
29.	28,000	·082+	•000	•017	•0221
30.	29,000	·082+	•000	•017	•0221
31.	30,000	.082	·000+	•017	•0221
32.	31,000	•082	•000	•017	.0221
33.	32,000	.082	•000	•018	•0234
34.	33,000	·081+	·001—	•018	.0234
35.	34,000	•081	·000+	•019	•0247
36.	35,000	·080+	·001—	•019	.0247
37.	36,000	•080	·000+	.020	•0260
38.	36,000+	Crushed.	·	•020	•0260

Table IV.—Holyhead-Rock Compression. Experiments D, Hard | Quartz. on Hard Quartz; pressure parallel to lamination. 1. 50 ·106 .000 .000 .0000 2. 3. 4. 1,000 ·106 .000 .000 .0000 2,000 ·106 .000 .000 .0000 .0000 3,000 ·106 .000 .000 4,000 5. ·106 .000 .000 .0000 6. .0052 5,000 .102 .004 .004 ·100+ 7. 6,000 .002-.004 .0052 .004 -00528. 7,000 ·100+ .000 ·100+ .000 .004 .0052 9. 8,000 ·100 .004 .0052 9,000 ·000+ 10. 10,000 .100 .000 .004 .0052 11. 11,000 ·100 .000 ·006 .0078 12. 12,000 ·098+ .002 -·006 .0078 13. 13,000 .008 .0104 14. ·098 ·000+ 14,000 .097 .001 .009 .0117 15. 15,000 ·096 .001 .010 .0130 16. 16,000 .093 .003 .013 ·0169 17. .0182 18. 17,000 .092 .001 .014 18,000 ·090+ .002-.014 .0182 19. .090 ·000+ .0208 19,000 .016 20. 21. 20,000 Crushed. .016 .0208

Table V.—Holyhead-Rock Compression. Experiments E, on Soft Slate; pressure transverse to lamination.



No. of experiment.	Pressure due to the unit of surface =1 square inch.	Compression readings of the column of 0.707 inch.	Compression readings due to the successive loads.	Total compressions produced by the load on column of 0.707 inch.	Total compressions reduced to a column of unit height = 1 inch.
	lbs.	in.	in.	in.	in.
1.	50	•088	•000	•000	.0000
2.	1,000	•087	•001	.001	.0014
3.	2,000	•086+	·001—	•001	.0014
4.	3,000	•086	·000+	.002	.0029
5.	4,000	.085	•001	.002	.0029
6.	5,000	•085	.000	•003	.0043
7.	6,000	•079	•006	•009	.0129
8.	7,000	·077+	.002-	.009	.0129
9.	8,000	·077+	•000	•009	.0129
10.	9,000	.077	·000+	•009	.0129
11.	10,000	•077	•000	•009	•0129
12.	11,000	.077	•000	•011	·0158
13.	12,000	.075	.002	.013	•0187
14.	13,000	•060	.015	.028	•0404
15.	14,000	•050	.010	.038	•0548
16.	15,000	Crushed.		.038	.0548

Note.—The cube E was 0.693 inch on the side, and the necessary reductions have been made in column 2 and subsequent ones.

Table VI.—Holyhead-Rock Compression. Experiments F, on Soft Slate; pressure parallel to lamination.



1.	50	.107	•000	.000	•0000
2.	1000	•105	.002	.002	.0029
3.	2000	·102+	·003—	.002	.0029
4.	3000	·102	+000	.002	.0029
5.	4000	•102	•000	.005	.0072
6.	5000	•099	•003	•008	.0115
7.	6000	•097	.002	•010	.0147
8.	7000	•089	•008	·018	.0259
9.	8000	•080	•009	.027	.0389
10.	8000+	Crushed.	-	.027	.0389

Note.—The cube F was 0.693 inch on the side, and the necessary reductions have been made in column 2 and subsequent ones.

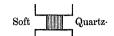
Table VII.—Holyhead-Rock Compression. Experiments G, on Soft Quartz; pressure transverse to lamination.



1.	50	•093	•006	•000	.0000	
2.	1,000	•093	•000	•000	.0000	
3.	2,000	•093	•000	•000	.0000	
4.	3,000	•090	•090	•003	.003	.0043
5.	4,000	·086+	·004—	.003	·0043 ·0043 ·0043	
6.	5,000	·086+	•000	.003		
7.	6,000	•086	·000+	•003		
8.	7,000	•086	•000	.007	.0101	
9.	8,000	·085+	·001	.007	.0101	
10.	9,000	·085+	•000	.007	.0101	
11.	10,000	•085	.085	•000 +	•008	.0115
12.	11,000	•084	•001	•009	.0129	
13.	12,000	•081	•003	.012	.0176	
14.	13,000	•068	•013	.025	.0359	
15.	14,000	•060	Crushed	before being full	ly loaded.	

Note.—The cube G was 0.694 inch on the side, and the necessary reductions have been made in column 2 and subsequent ones.

Table VIII.—Holyhead-Rock Compression. Experiments H, on Soft Quartz; pressure parallel to lamination.



No. of experiment.	Pressure due to the unit of surface =1 square inch.	Compression readings of the column of 0.707 inch.	Compression readings due to the successive loads.		Total compressions reduced to a column of unit height =1 inch.
	lbs.	in.	in.	in.	in.
1.	50	.170	•000	.000	.0000
2.	1,000	•144	.026	.026	.0374
3.	2,000	·101+	·043—	.069	.0992
4.	3,000	.101	·000+	•069	.0993
5.	4,000	·100	•001	.070	.1007
6.	5,000	• • • • • • • • • • • • • • • • • • • •	•001	.071	.1021
7.	6,000	.098	.001	.072	.1036
8.	7,000	.049	•049	121	.1741
9.	7,000 +	Crush	ed before the inc	creased load was a	pplied.

Note.—The cube H was 0.695 inch on the side, and the necessary reductions have been made in column 2 and subsequent ones.

Table IX.—Holyhead-Rock Compression. Slate Rock.—Results of compression compared. Column of unit length =1 inch.

	1		0		
No. of experiment.			B. Hard slate with the lamina.	E. Soft slate across lamina.	F. Soft slate with the lamina.
	lbs.	in.	in.	in.	in.
1.	50	.0000	.0000	.0000	•0000
2.	1,000	.0052	.0130	.0014	.0029
3.	2,000		•0390		
4.	3,000	••••	.0403	.0029	
5.	4,000	••••	•0416	•••••	.0072
6.	5,000	·0091		.0043	.0115
7.	6,000	•••••		.0129	.0147
8.	7,000	.0104	•0468	••••	.0259
9.	8,000		•0494		.0389
10.	9,000	*****		*****	Crushed.
11.	10,000		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		
12.	11,000	•••••		.0158	
13.	12,000	.0117		·0187	
14.	13,000			.0404	
15.	14,000			.0548	
16.	15,000		.0520	Crushed.	
17.	16,000	•••••	.0533		
18.	17,000	•••••	.0572		
19.	18,000		.0585		
20.	19,000	•0130	0000		
21.	20,000	0150			
22.	21,000				
23.	22,000				
24.	23,000	•0143			
25.	24,000	Crushed.	.0624		
26.	25,000	Crusheu.	0024		
27.	26,000		•0650		
28.	27,000		·0689		
29.	28,000		Crushed.		
30.	29,000		O usucu.		
	ssion for each { unit of surface {	in. •0006217 up to 23,000 lbs.	in. •0025000 up to 26,000 lbs.	in. •0039144 up to 14,000 lbs.	in. •0037000 up to 7000 lbs.

Table X.—Holyhead-Rock Compression. Quartz Rock.—Results of Compression compared. Column of unit length =1 inch.

No. of experiment.	Pressure in pounds on unit of surface = 1 square inch.	C. Hard quartz across lamina.	D. Hard quartz with the lamina.	G. Soft quartz across lamina.	H. Soft quartz with the lamina.
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37.	lbs. 50 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 10,000 11,000 12,000 13,000 14,000 15,000 16,000 17,000 18,000 20,000 21,000 22,000 23,000 24,000 25,000 26,000 27,000 28,000 29,000 30,000 31,000 32,000 33,000 34,000 35,000 36,000	in0000 -00390065 -007800910104 -0117 -0156022102340247	in0000005200580104 -0117 -0130 -0169 -0182 -0208 Crushed.	in. •0000 •0043 •0101 •0115 •0129 •0176 •0359 Crushed.	in. •0000 •0374 •0992 •0993 •1007 •1021 •1036 •1741 Crushed.
38.	37,000	Crushed.	·		
Mean compress 1000 lbs. on un	sion for each it of surface.	in. •0007085 up to 35,000 lbs.	in. •0010947 up to 19,000 lbs.	in. •0014666 up to 12,000 lbs.	in. •0172666 up to 6000 lbs.

An examination of these Tables presents some remarkable and, so far as I am aware, now for the first time observed results.

As might have been expected, the quartz-rock is much less compressible generally than the slate-rock, with this exception, however, that the softest specimens of quartz-rock, and those alone, are much more compressible than the softest slate, when both are compressed in the direction of or parallel to the lamination.

In this direction of compression, the hardest slate is more than double as compressible as the hardest quartz.

When compressed transverse to the lamina, however, the hard slate and hard quartz prove to have very nearly the same coefficient of compressibility, which is very small for both; while the softest slate and the softest quartz, compressed in the same way (transverse to lamina), have also nearly the same coefficient of compressibility, but one about four times as great as for the hardest like rocks.

These facts point towards the circumstance of the original deposit and formation of these rocks as their efficient causes. Both rocks consist of particles more or less wedge-shaped and flat, angular fragments more or less crystalline, deposited together, with their larger dimensions in the planes of lamination, which lamination has been produced by enormous compression in a direction transverse to its planes. Hence the mass of these rocks has already been subjected to enormous compression in the same direction as that in which we now find their further compressibility the least. But, besides that we might from this cause alone anticipate a higher compressibility when the pressure is applied to them parallel to the lamination, another condition comes into play; their aggregation of flat, wedge-shaped particles, when thus pressed edgeways, tends powerfully to their mutual lateral expansion, and hence to their giving way in the line of pressure.

The per-saltum way in which all the specimens of both rocks yield, in whatever direction pressed, is another noteworthy circumstance. On examining the Tables I. to VIII. it will be seen that the compressions do not constantly advance with the pressure, but that, on the contrary, the rock occasionally suffers almost no sensible compression for several successive increments of pressure, and then gives way all at once (though without having lost cohesion, or having its elasticity permanently impaired) and compresses thence more or less for three or four or more successive increments of pressure, and then holds fast again, and so on. This phenomenon is probably due to the mass of the rock being made up of intermixed particles of several different simple minerals, having each specific differences of hardness, cohesion, and mutual adhesion, and which are, in the order of their resistances to pressure, in succession broken down, before the final disruption of the whole mass (weakened by these minute internal dislocations) takes place.

Thus it would appear that the micaceous plates and aluminous clay-particles interspersed through the mass give way first. The chlorite in the slate, and probably felsparcrystals in the quartz-rock, next, and so on in order, until finally the elastic skeleton of silex gives way, and the rock is crushed. It is observable, also, that this successive disintegration does not occur at equal pressures, in the same quality and kind of rock, when compressed transverse and parallel to the lamination. It follows from this constitution of these (and probably of all) rocks that very different powers of transmitting wave-impulses must arise when the originating forces vary considerably in amount of primary compression. It is almost superfluous also to point out the great differences in wave-transmissive power in directions transverse and parallel to lamination that these

experiments disclose. They prove to us that, in an earthquake shock of given original power, the vibrations will have the largest amplitude when transmitted in the line of the lamination, but may be propagated with the greatest velocity in directions transverse to the same, assuming in both cases the rock solid and unshattered.

In the following Table XI, the general results are deduced, and the *mean campressions* for each of the rocks calculated, and finally the moduli of elasticity are obtained in pounds and in feet; the specific gravities adopted in calculating the latter being those given in the body of the paper, as follows:—

									Weight of a prism 1 foot long and 1 inch square.
								sp. gr.	lbs.
Hardest slate .								2.763	1.1992
Softest slate			•				•	2.746	1.1918
Hardest quartz .								2.656	1.1528
Softest quartz .								2.653	1.1515
Mean for slate .									1.1955
Mean for quartz					•			2.6545	$1 \cdot 1522$
General mean for	· bo	oth	\mathbf{roc}	ks				2.7045	1.1739

The load on the unit of surface (1 square inch) at which the elastic limit of the rock is passed, and that at which it is finally crushed, together with the modulus of cohesion or resistance to compression, are also given, and will be useful to the engineer and architect. In the last column the value of my own modification of Poncelet's coefficient T, (la force vive de rupture) is calculated in foot pounds, and represents the relative work done at fracture in each case.

Table XI.—Holyhead-Rock Compression.

General results reduced, Modulus of Cohesion and of Elasticity, &c.—Slate and Quartz.

Class of Rock, ar	Slate, hardest, across laminat Quartz, hardest, across lamin Slate, hardest, parallel to lar Quartz, hardest, parallel to la	Slate, softest, ac Quartz, softest, Slate, softest, po Quartz, softest,	Slate, mean for Quartz, mean for Slate, mean for Quartz, mean f	Calculated me Slate, hard and soft, mean 9 & 11)	14. Quartz, hard and soft, mean 10 & 12)	in both direc
Class of Rock, and direction of pressure in relation to structure.	Slate, hardest, across lamination Quartz, hardest, across lamination Slate, hardest, parallel to lamination Quartz, hardest, parallel to lamination	Slate, softest, across lamination	Slate, mean for hard and soft, aeross lamination	Calculated mean of means. d soft, mean for both directions (Nos.)	uartz, hard and soft, mean for both directions (Nos.) 10 & 12) 20 and 10	
Coefficient of compression on unit surface for 1000 lbs.	in. •0006217 •0007085 •0025000 •0010947	.0039144 .0014666 .0037000 .0172666	.0022680 .0010875 .0031000	.0026840	.0051340	06068000
Elastic limit for com-	1bs. 22,000 32,000 18,000 17,000	12,000 11,000 6,000 7,000	17,000 16,500 12,000 12,000	14,500	16,750	15,625
Crushing load on the unit of surface.	1bs. 24,000 37,000 27,000 20,000	15,000 14,000 9,000 8,000	19,500 25,500 18,000 14,000	18,750	19,750	19,250
Modulus of cohesion (com- pression).	ft. 20,014 32,095 22,515 17,349	12,586 12,158 7,552 6,948	16,311 22,132 15,056 12,151	15,684	17,141	16,398
Modulus of elasticity.	lbs. 8,042,464 7,057,163 2,000,000 4,567,461	1,277,335 3,409,246 1,351,351 289,576	2,204,585 4,597,701 1,612,903 544,627	1,862,880	973,899	1,279,099
Modulus of elasticity. L.	ft. 6,706,524 6,121,758 1,667,778 3,962,013	1,071,769 2,960,699 1,133,874 251,477	1,844,069 3,990,455 1,349,145 472,684	1,566,541	845,252	1,089,615
Coefficient. Tr.	ft. lbs. 1.2432 2.1830 5.6241 1.8240	4.8930 1.7108 2.7747 11.6112	3.6855 2.3103 4.6494 10.7100	4.1914	8.4490	6.5097

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To apply the results thus obtained to those of experimental wave-transmission at Holyhead.

Poisson has shown* that the velocity of wave-transmission (sound) in longitudinal vibrations of elastic prisms is

When g has its usual relation to gravity, l and p are the length and weight of the prism, and $q = \frac{\Delta}{\delta}$, Δ being a weight that is capable of elongating the prism by an amount $=\delta l$, or extending it to the length

$$l(1+\delta)$$
.

Substituting, we have

$$V^2 = \frac{gl\Delta}{p\delta};$$

but

$$\Delta$$
: W:: δ :1,

W being the weight capable of doubling the length of the prism. Therefore

$$V^2 = \frac{glW\delta}{p\delta} = \frac{glL}{l} = gL,$$

or

$$V = \sqrt{gL}$$
. (II.)

So that L being the modulus of elasticity of the solid, expressed in feet, the velocity of wave-transmission through it, if absolutely homogeneous and unbroken, is

$$V=5.674\sqrt{L}$$
. (III.)

Where, owing to want of homogeneity, or to shattering, or other such condition, as found in natural rock, the experimental value of V differs from the above theoretic one, we may still express the former by the same general form of equation—

$$V'=\alpha\sqrt{L},\ldots$$
 (IV.)

in which the coefficient α expresses the ratio to \sqrt{g} that the actual or experimental bears to the theoretic (or maximum possible) velocity of wave-transmission.

In the slate- and quartz-rocks of Holyhead, I ascertained the mean *lowest* velocity of wave-transmission (for small explosions or impulses) to be 1089 feet per second (omitting decimals), the mean *highest* velocity 1352 feet per second, and the *general mean* velocity from all, 1220 feet per second.

Applying equation (IV.) to these numbers, and adopting the values of L given in Table XI. (mean of Nos. 9 and 10), we obtain

$$\alpha = \frac{V'}{\sqrt{L}};$$

^{*} Traité de Mécanique, vol. ii. p. 319.

and for the three preceding velocities, α has the following values:—

ft. per sec.

$$1 \dots V' = 1089 \dots \alpha = \frac{1089}{\sqrt{2917262}} = \frac{1089}{1708} = 0.637$$

$$2 \dots V' = 1352 \dots \alpha = \frac{1352}{\sqrt{2917262}} = \frac{1352}{1708} = 0.791$$

$$3 \dots V' = 1220 \dots \alpha = \frac{1220}{\sqrt{2917262}} = \frac{1220}{1708} = 0.714$$

The actual velocity of wave-transmission in the slate and quartz together, therefore, was to the theoretic velocity due to the solid material as

$$\alpha: \sqrt{g} \text{ or } 0.714:5.774, \text{ or } 1.00:7.946.$$

From which it results that *nearly seven-eighths* of the full velocity of wave-transmission due to the material is lost by reason of the heterogeneity and discontinuity or shattering of the rocky mass, as it is found piled together in nature.

This loss would be larger with still smaller originating impulses, and *vice versâ*, but in what proportion we are not at present in a position to know.

If we may for a moment allude to final causes, we cannot but be struck with this beneficent result (amongst others) arising from the shattered and broken-up condition of all the rocky masses forming the habitable surface of our globe,—that the otherwise enormous transit-velocity of the wave-form in earthquake shocks is by this simple means so reduced.

That this retardation is mainly effected by the multiplied subdivisions of the rock, and in a very minor degree by differences in the elastic moduli of rock of different species, is apparent on examining the Tables IV. and V. of the previous part of this Report referring to the experiments at Holyhead.

Although, therefore, we are now enabled, from what precedes, to calculate values for α , for the slate rocks and for the quartz of Holyhead, separately, and thus obtain separate values for V', for *each* of those rocks; the result would probably be more or less delusive, as we have no possible means of deciding what is the relative amount of shattering and discontinuity, for equal horizontal distances, in each of these two rocks, nor what the relative retarding powers of planes of separation running in variable directions, and at all possible angles across the line of wave-transit, as compared with their retarding powers if either all transverse to, or all in the same direction as, the wave-path.

The greatest possible mean velocity of wave-propagation, in rock as perfectly solid and unshattered as our experimental cubes, is determinable for both slate and quartz in the two directions of transmission, viz. transverse and in the line of lamination, from equation (III.), and the mean values of L in Nos. 9 and 10, and 11 and 12, Table XI., as follows:—

Mean of slate and quartz transverse to lamination ... $V=5.674\sqrt{2917262}=9691$ Mean of slate and quartz in line of lamination $V=5.674\sqrt{910914}=5415$ This great difference of velocity, due to the difference in the molecular properties of the material of the rocks in their opposite directions, is, as our Holyhead experiments prove, almost wholly obliterated by the vastly increased degree of discontinuity and shattering, in the directions approaching that of lamination, or transverse to the wavepath in the first case.

It is necessary to guard against any misconception as to the import of this result. The fact ascertained and just enunciated is this, that the velocity of wave-transmission is greater in the material of these rocks in a direction across their lamination than in one longitudinal to the same, provided or assuming the material be perfectly unshattered in both—as homogeneous, in fact, as the small specimen-cubes experimented upon. And were the whole mass of the rock, as it lies in its mountain-bed, as homogeneous as such cubes, then the velocity of wave-transmission would actually be greater across long ranges of natural lamination, than edgeways to them. The opposite, however, is often the case; the wave-transit period is slower as the range of rocky mass is more shattered, discontinuous, and dislocated.

These conditions affect rocks in nature most in or about their planes of bedding, lamination, &c., and hence most retard wave-impulses transverse to these planes; so that the more rapid wave-transmissive power of the material of the rock in a direction transverse to the lamination may be more than counterbalanced by the discontinuity of its mass transverse to the same direction.

The results of Wertheim, on the transmission of sound in timber, proved the velocity to be greatest in a direction longitudinal to the fibres and annual rings of wood; less in a direction perpendicular to the same, and from the centre of the tree radially towards its exterior; and least of all in a direction, quam prox., parallel to the annual rings, and perpendicular to the longitudinal fibres; that is to say, that in each case the velocity of sound was rapid in proportion to the less compressibility of the wood in the same direc-His results might seem at first to conflict with those which I have announced. Any such conclusion, however, would be a mistake; on the contrary, my results perfectly analogize with those above alluded to. The difference between the cases is, that wood in mass, however large, is practically homogeneous and unshattered, and that its direction of least compressibility is longitudinal to its lamina (or annual rings); whereas the direction of least compressibility of rock is transverse to its lamina (which have been already powerfully compressed in this direction). In fact, as respects the point here in question, there is no true analogy in structure between the lamination (by annual rings) of wood, and the lamination or bedding of rock.

It follows from what precedes, that earthquakes and rocks as both actually occur in nature—the rocks being of a stratified or laminated form (generally all sedimentary rocks)—must present the following conditions as to rate of transit of shock:—

1st. If such rocks were perfectly unshattered, and the beds or laminæ in absolute contact, the shock would be transmitted more rapidly across these than in their own direction.

The difference is more in favour of the transverse line, in proportion as the rock is made up more of angular sedimentary particles of very unequal dimensions, the longest being parallel to the general lamination, and in proportion as the imbedding paste is softer in relation to such particles.

Some sedimentary rocks no doubt exist, made up of particles perfectly uniform and equal in all three dimensions, and without imbedding paste—such as the lithographic stones of Germany, the Apennine marl-beds, &c., in which (assuming the above condition as to continuity) the transit-period would probably be alike in all directions.

2nd. The actual amount of shattering and discontinuity in nature being usually greatest, upon the whole, in planes parallel to bedding or lamination, the transit-rate of shock is most generally fastest in the line of the beds or lamination, rather than across them.

Or, at least, this latter condition may interfere with the former to the extent of partial, complete, or more than complete obliteration.

I am not aware that experiments have previously been made at all upon the compressibility, &c. of the slate- and quartz-rocks of Holyhead; and as these rocks are being employed there upon a vast scale for submarine building works, it may not be out of place to draw a few conclusions of a character useful to the practical engineer from the data that have been obtained. Some conclusions may be drawn which are applicable to all classes of laminated rocks in the hands of the engineer.

It is a very prevalent belief that slate-rock (for example), in the form of the sawed roofing-slate of Anglesea or of Valentia (Ireland), will bear a much greater compressive load when the pressure is in the direction of the laminæ, than in one across them. This the preceding experiments prove to be wholly a mistake—one that has very probably arisen from some vague notion of an analogy with timber compressed the end-way of the grain.

It is now certain that Silurian slates and quartz-rock, and probably all sedimentary laminated rocks, whether with cleavage or not, are much weaker to resist a crushing force edgeways to the lamina, than across the same, and that the range of compressibility is much greater, for equal loads, in the former direction.

The fact now ascertained, as to the great relative compressibility of laminated rock in the direction of the laminæ, also points out the reason of the great bearing-power to sustain impulsive loads, which the toughest and most cohesive examples of slate-rocks, such as the slates of Caernarvonshire, present; for there can be no grounds to doubt that the high compressibility of rocks of this structure in the plane of the lamina is also accompanied with a high coefficient of extensibility, although probably confined within much narrower limits as to incipient injury to perfect continuity.

My experiments point out that the Silurian slate of Holyhead (the mean both of the hard and the soft) is crushed by a load applied across the lamina of about 1250 tons per square foot, and that its molecular arrangement is permanently injured at a little more than 1000 tons per square foot.

4 z

The quartz-rock (the mean of both hard and soft) is crushed by a load, applied in the same manner, of 1630 tons per square foot, and its molecular arrangement is permanently injured at less than 1000 tons per square foot. The quartz-rock gives the highest measure of ultimate resistance, but it is the less trustworthy material when loaded heavily.

Neither of these sorts of rock, if loaded so as to be pressed in the direction of the lamina, would sustain more than about 0.7 of the above loads at the crushing-point and at that of permanent injury, respectively. From the extreme inequality found within narrow limits in both rocks as quarried, neither should be trusted for safe load in practice with more than about $\frac{1}{20}$ th of the mean load that impairs their molecular arrangement, as ascertained from selected specimens, or (say) not to more than 50 tons per square foot for passive or 25 tons per square foot for impulsive loads.

The high relative compressibility of laminated rocks in the direction of the lamina might probably be made advantageous use of, where they are employed as a building material, for the construction of revetment or other walls of batteries exposed to the stroke of cannon-shot, by building the work (under suitable arrangements to obviate splitting up) with the planes of the laminæ in the direction of the line of fire, *i. e.* perpendicular to the faces of the work; for on inspecting the last column in Table XI., which contains the values of T, under the several conditions of rock and of compression, it is at once apparent how much greater is the work done in crushing the slates and the quartz in their toughest and most compressible direction, *i. e.* in the direction of the laminæ,—twice as much work being, upon the average, consumed in crushing the rock in this direction as suffices to destroy its coherence in the one transverse to the laminæ, and the difference in the two, in the case of the softest quartz (Nos. 6 & 8), being as much as about 5 to 1.

It would be unsuitable, however, to the present memoir here to pursue further such practical deductions suggested by the results experimentally obtained.